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Research Paper

Buys-Ballot Estimates of Quadratic Trend Component and Seasonal Indices and Effect of Incomplete Data in Time Series

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ABSTRACT

This study examines the Buys-Ballot estimates of quadratic trend component and seasonal indices and effect of incomplete data in descriptive time series analysis. This paper is to provide estimates of trend parameters and seasonal indices using Buys-Ballot table with incomplete observations. The methods adopted in this study are 1) Mean imputations 2) Regression imputation 3) Buys-Ballot table for time series decomposition. The methods are based on the row, column and overall means of time series data arranged in Buys-Ballot table with m rows and s columns. The model structure is additive.

Keywords: Incomplete Data, Trend Parameter, Seasonal Indices, Quadratic Trend Cycle Component, Additive Model, Buys-Ballot Table.

1 INTRODUCTION

Time series application has become very important in making decision in various fields. In providing solutions to real life problems involving time series, most existing statistical methods in solving these problems deals with observed values throughout the data. Series with missing observations cannot be solved using these methods. Missing observations occurred because of several problems, such as technical fault or human errors (the object of observation did not give enough data to the observer). This study takes into consideration the effect of incomplete observations on Buys-Ballot estimates when trend cycle component of time series is quadratic.

Okereke, *et al* [1] in a study on the chain base, fixed base and classical methods of time series decomposition with the cubic trend component with emphasis on additive model observed that the chain base method are both used for time series decomposition, the recommended chain base method when a case of multicollinearity in a time series model. Ljung [2] obtained an expression for likelihood function of the parameters in an autoregressive moving average (AMA) model when there are missing observations within the time series data. Luceno [3] extended the method by Ljung [2] for estimating missing observations and evaluating the corresponding likelihood function in scalar time series to the vector cases.

Cyclical component which is regarded as long oscillations appears to appreciable magnitude only in long period of time. However, if short period of time are involved, the trend is jointly combined into cyclical component. Chatfield, [4] and the observed time series $\left(X_t, t = 1, 2, ..., n\right)$ can be decomposed into the trend-cycle component $\left(M_t\right)$, seasonal component $\left(S_t\right)$ and the irregular/residual component $\left(e_t\right)$. Therefore, the decomposition models are

Additive Model

$X_t = M_t + S_t + e_t$ (1)

Multiplicative Model:

$X_t = M_t \times S_t \times e_t$ (2) and Mixed Model

$X_t = M_t \times S_t + e_t$. (3)

This study is limited to time series data with quadratic trend and admits additive model using registered number road traffic accidents over the period 2009 to 2018. The emphasis is to estimate the trend parameters and seasonal indices using Buys-Ballot table with incomplete observations which takes into consideration additive model structure and quadratic trending curve. Empirical example was used in this study and data was drawn from monthly records of number of road traffic accidents from the Federal Road Safety Office in Owerri, Imo State, Nigeria over the period of January, 2009 to December, 2018. One hundred and twenty (120) registered road traffic accidents were considered over the period under investigation in which three (3) registered accidents were not accounted for. The observed series was transformed and the trend parameters and seasonal indices estimated using Regression imputation method. The missing observations will be estimated using this decomposition method and the entire process of estimation will be repeated without missing observations. The estimated trend parameters and seasonal indices with and without missing observations were therefore compared

2. MATERIALS AND METHODS

2.1Buys-Ballot Procedure for Time Series Decomposition

This method is based on the row, column and overall means of time series data arranged in a Buys-Ballot table with m rows and s columns, m is the number of observations in each column and s is the number of columns. For details of Buys-Ballot procedure, see Iwueze and Nwogu, [5-7] Iwueze and Ohakwe, [8] Dozie, [9] Dozie, *et al*, [10] Dozie and Ijomah. [11]

Table 1: Buys - Ballots table for seasonal time series

Rows/ Period (i)	Columns (season) j								
	1	2	⋯	j	...	s	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
1	X_1	X_2	⋯	X_j	⋯	X_s	$T_{1.}$	$\bar{X}_{1.}$	$\hat{\sigma}_1$
2	X_{s+1}	X_{s+2}	⋯	X_{s+j}	⋯	X_{2s}	$T_{2.}$	$\bar{X}_{2.}$	$\hat{\sigma}_2$
3	X_{2s+1}	X_{2s+2}	⋯	X_{2s+j}	⋯	X_{3s}	$T_{3.}$	$\bar{X}_{3.}$	$\hat{\sigma}_3$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
i	$X_{(i-1)s+1}$	$X_{(i-1)s+2}$	⋯	$X_{(i-1)s+j}$	⋯	$X_{(i-1)s+s}$	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
m	$X_{(m-1)s+1}$	$X_{(m-1)s+2}$	⋯	$X_{(m-1)s+j}$	⋯	X_{ms}	$T_{m.}$	$\bar{X}_{m.}$	$\hat{\sigma}_{m.}$
$T_{.j}$	$T_{.1}$	$T_{.2}$	⋯	$T_{.j}$	⋯	$T_{.s}$	$T_{..}$		
$\bar{X}_{.j}$	$\bar{X}_{.1}$	$\bar{X}_{.2}$	⋯	$\bar{X}_{.j}$	⋯	$\bar{X}_{.s}$		$\bar{X}_{..}$	
$\hat{\sigma}_{.j}$	$\hat{\sigma}_{.1}$	$\hat{\sigma}_{.2}$	⋯	$\hat{\sigma}_{.j}$	⋯	$\hat{\sigma}_{.s}$			$\hat{\sigma}_{..}$

In this arrangement each time period t is represented in terms of the period i (e.g. year) and season j (e.g. month of the year), as $t = (i-1)s + j$. Thus, the period (row), season (column) and overall totals, means and variances are defined as

$$T_{i.} = \sum_{j=1}^{s} X_{(i-1)s+j}, \quad \bar{X}_{i.} = \frac{T_{i.}}{s}, \quad \hat{\sigma}_{i.}^2 = \frac{1}{s-1}\sum_{j=1}^{s}\left(X_{ij} - \bar{X}_{i.}\right)^2$$

$$T_{.j} = \sum_{i=1}^{m} X_{(i-1)s+j}, \quad \bar{X}_{.j} = \frac{T_{.j}}{m}, \quad \hat{\sigma}_{.j}^2 = \frac{1}{m-1}\sum_{i=1}^{m}\left(X_{ij} - \bar{X}_{.j}\right)^2$$

$$T_{..} = \sum_{i=1}^{m}\sum_{j=1}^{s} X_{(i-1)+j}, \quad \bar{X}_{..} = \frac{T_{..}}{n}, n = ms, \quad \hat{\sigma}_{x}^2 = \frac{1}{n-1}\sum_{i=1}^{m}\sum_{j=1}^{s}\left(X_{ij} - \bar{X}_{..}\right)^2$$

2.2Mean Imputation (MI)

This is one of the methods of replacing missing observations. The mean imputation method replaces the missing observation with the mean of the values before the missing position. This is done by taking the summation of the observations and dividing by the number of values before the missing position.

$$MI = \hat{X}_{(i-1)s+j} = \frac{1}{(i-1)s+j-1}\left[X_1 + X_2 + X_3 + \ldots + X_{(i-1)s+j}\right] = \frac{1}{n}\sum_{i=1}^{n} X_i \quad (4)$$

2.3Regression Imputation (RI)

Regression imputation method is one of the best method of estimating missing observation. This method estimates the missing observation by the estimate of the trend at the point of the missing observation. Therefore, if the remaining observations of the series are used to determine estimates of the trend parameters and seasonal indices, then the estimates of the missing value at $(i-1)s+j$ of the trend-cycle component of the regression imputation method for the quadratic is given as

$$\hat{X}_{(i-1)s+j} = \hat{a} + \hat{b}[(i-1)s+j] + \hat{c}[(i-1)s+j]^2 \quad (5)$$

The model of the regression imputation methods are given by;

For Additive model

$$\hat{X}_{(i-1)s+j} = \hat{M}_{(i-1)s+j} + \hat{S}_{(i-1)s+j} \quad (6)$$

For multiplicative model

$$\hat{X}_{(i-1)s+j} = \hat{M}_{(i-1)s+j} \times \hat{S}_{(i-1)s+j} \quad (7)$$

For mixed model

$$\hat{X}_{(i-1)s+j} = \hat{M}_{(i-1)s+j} \times \hat{S}_{(i-1)s+j} \quad (8)$$

2.4Estimation of Trend Parameters and Seasonal Indices

The expression of the quadratic trend is given by

$$\bar{X}_i = a + bt + ct^2 \quad (9)$$

Iwueze and Nwogu [7] gave the estimation of the trend and seasonal indices for an additive model when trend-cycle component is quadratic as;

$$\hat{a} = a^{\prime} + \left(\frac{s-1}{2}\right)\hat{b} - \left(\frac{(s-1)(2s-1)}{6}\right)\hat{c} \quad (10)$$

$$\hat{b} = \frac{b^{\prime}}{s} + \hat{c}(s-1) \quad (11)$$

$$\hat{c} = \frac{c^{\prime}}{s^2} \quad (12)$$

$$\hat{S}_j = \bar{X}_{.j} - d_j \quad (13)$$

$$d_j = \hat{a} + \frac{\hat{b}}{2}(n-s) + \frac{\hat{c}(n-s)(2n-s)}{6} + \hat{b} + \hat{c}(n-s)j + \hat{c}j^2 \quad (14)$$

2.5Estimation of Missing observation in the Transformed and Untransformed Time Series Data

The transformed estimated missing observation of the estimated trend parameters and seasonal indices for additive model is given by;

$$\hat{X}_{ij} = \hat{a} + \hat{b}[(i-1)s+j] + \hat{c}[(i-1)s+j]^2 + \hat{S}_j \quad (15)$$

Hence, the exponent of the estimated observation in the transformed series gives the estimate of the missing observation in the untransformed series.

$$Untransfomed \quad \hat{X}_{ij} = e^{\hat{a} + \hat{b}[(i-1)s+j] + \hat{c}[(i-1)s+j]^2 + \hat{S}_j} \quad (16)$$

3. Analysis:

This section discusses real life databased on monthly data on road traffic accident for the period of ten (10) years. One hundred and seventeen (117) registered road traffic accidents were considered from January to December 2009 to 2018 in which three (3) registered accidents were not accounted for. The time plots of actual and transformed series with missing data are given in figure 3.1 and 3.2

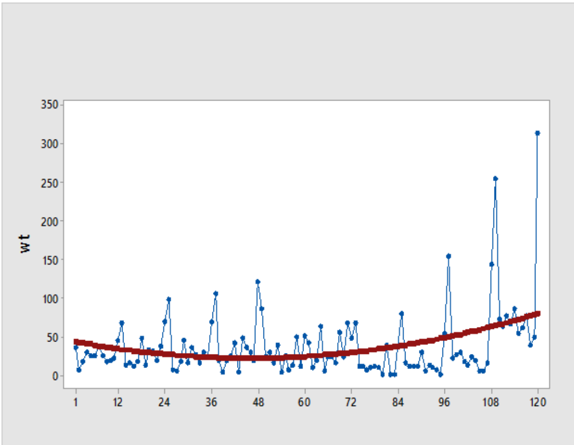

Figure 3.1: Time plot of the actual series on the number of road accidents between (2009-2018)

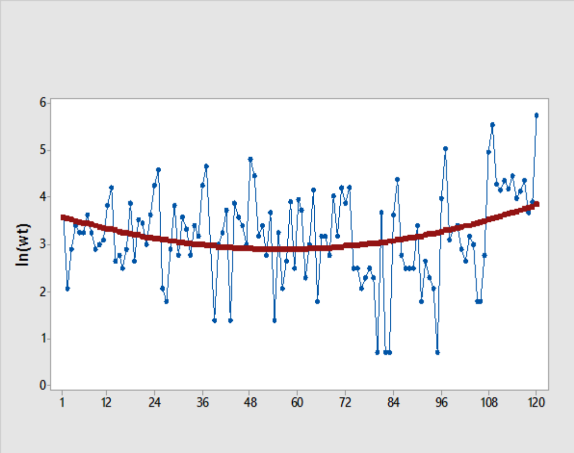

Figure 3.2: Time plot of the transformed series on the number of road accidents between (2009-2018)

Using (10),(11) and (12) we obtain

$$\hat{c} = \frac{0.0393}{144} = 0.0003$$

$$\hat{b} = \frac{-0.402}{12} + 0.0003(12-1) = -0.0302$$

$$\hat{a} = 3.885 + \left(\frac{12-1}{2}\right)(-0.0302) - \left(\frac{(12-1)(24-1)0.0003}{6}\right) = 3.7062$$

Using (13)

$$d_j = 3.7062 + (-0.0151)(120-12) + \frac{0.\hat{0}003(120-12)(240-12)}{6} + (-0.0302) + 0.0003(120-12)j + 0.0003j^2$$

$$\bar{X}._j - (3.2764 + 0.0324j + 0.0003j^2)$$

j	$\bar{X}._j$	$\hat{S}_j$	$Adj\ \hat{S}_j$
1	4.442	1.1329	1.4410
2	2.790	-0.5524	-0.2443
3	2.770	-0.6063	-0.2983
4	3.1393	-0.2715	-0.0366
5	3.053	-0.3879	-0.0798
6	3.118	-0.3636	-0.0555
7	2.894	-0.6239	-0.3158
8	2.929	-0.6258	-0.3177
9	3.092	-0.5003	-0.1922
10	2.814	-0.8164	-0.5083
11	2.9983	-0.7032	-0.3951
12	4.330	0.6216	0.9294

3.1 Buys –Ballot Estimates of Trend and Seasonal Indices with Missing Observations

The quadratic trend of the row averages in the Buys-Ballot with missing observations is given as;

$$\bar{X}_i = 3.885 - 0.402t + 0.0393t^2 \quad (17)$$

Where,

$$a^| = 30885,\ b^| = -0.402,\ c^| = 0.0393$$

Here, the row averages in the Buys-Ballot table without missing observations of the quadratic trend is given as;

$$\bar{X}_i = 3.858 - 0..383t + 0.0377t^2 \quad (18)$$

Using (10),(11) and (12) we obtain

$$\hat{c} = \frac{0.0377}{144} = 0.0003$$

$$\hat{b} = \frac{-0.383}{12} + 0.0003(12-1) = -0.0286$$

$$\hat{a} = 3.858 + \left(\frac{12-1}{2}\right)(-0.0286) - \left(\frac{(12-1)(24-1)0.0003}{6}\right) = 3.6881$$

Using (13)

3.2 Buys-Ballot Estimates of Trend and Seasonal Indices without Missing Observations

$$d_j = 3.6881 + \frac{(-0.0286)}{2}(120-12) + \frac{0.\hat{0}003(120-12)(240-12)}{6} + (-0.0286) + 0.0003(120-12)j + 0.0003j^2$$

Therefore,

$$\bar{X}_j - (3.3463 + 0.0324j + 0.0003j^2)$$

j	$\bar{X}_{.j}$	$\hat{S}_j$	$Adj\ \hat{S}_j$
1	4.442	1.063	1.4346
2	2.790	-0.6223	-0.2507
3	2.770	-0.6762	-0.3046
4	3.210	-0.2707	0.1009
5	3.058	-0.4578	-0.0862
6	3.118	-0.4335	0.0619
7	2.894	-0.6938	-0.3222
8	2.929	-0.6957	-0.3241
9	3.066	-0.5962	-0.2246
10	2.814	-0.8863	-0.5147
11	2.998	-0.741	-0.3694
12	4.330	0.5517	0.9229

Table 2: Row totals, means and variances with missing observations.

Periods i	With missing observations			
	r_i	T_i	$\bar{X}_i$	$\hat{\sigma}_i^2$
1	10	31.810	3.1810	0.2070
2	10	32.830	3.2830	0.3940
3	10	32.050	3.2050	0.6650
4	10	32.570	3.2570	1.1190
5	9	31.303	3.1303	0.7600
6	10	32.840	3.2840	0.5740
7	9	21.887	2.1887	1.3910
8	9	28.011	2.8011	0.9510
9	10	31.581	3.1580	1.0140
10	10	44.00	4.4000	0.3840
Overall Total		31.8882	3.18882	0.7459

$$n = \sum_{j=1}^{r} c_j = \sum_{i=1}^{c} r_i = total\ number\ of\ observation$$

Where,

r_i = Number of observation in the r^th row

c_j = Number of observation in the j^th column.

Table3: Row totals, means and variances without missing observations.

Periods i		Without missing observations		
	r_i	T_i	$\bar{X}_i$	$\hat{\sigma}_i^2$
1	10	31.8	3.18	0.21
2	10	32.8	3.28	0.39
3	10	32.1	3.21	0.67
4	10	32.6	3.26	1.12
5	10	31.9	3.19	0.79
6	10	32.8	3.28	0.57
7	10	22.4	2.24	1.24
8	10	28.2	2.82	0.58
9	10	31.6	3.16	1.01
10	10	44.0	4.40	0.38
Overall Total	100	32.0	3.20	0.75

Table 4: Column totals, means and variances with missing observations

Seasons j		With missing observations		
	c_j	$T_{.j}$	$\bar{X}_{.j}$	$\hat{\sigma}_{.j}^2$
1	12	53.30	4.44	0.33
2	12	33.48	2.79	0.43
3	12	33.24	2.77	0.64
4	11	34.53	3.14	0.54
5	12	36.70	3.06	0.55
6	12	37.42	3.12	0.73
7	12	34.73	2.89	0.71
8	12	35.15	2.93	0.98
9	11	34.01	3.09	0.65
10	12	33.77	2.81	0.99
11	11	32.98	3.00	1.47
12	12	51.96	4.33	0.43
Overall Total	141	37.61	3.20	0.70

Table 5: Column totals, means and variances without missing observations.

Seasons j		Without missing observations		
	c_j	$T_{.j}$	$\bar{X}_{.j}$	$\hat{\sigma}_{.j}^2$
1	12	53.28	4.44	0.33
2	12	33.48	2.79	0.43
3	12	33.24	2.77	0.64
4	12	38.52	3.21	0.58
5	12	36.72	3.06	0.55
6	12	37.44	3.12	0.73
7	12	34.68	2.89	0.71
8	12	34.16	2.93	0.98
9	12	36.84	3.07	0.62
10	12	33.72	2.81	0.99
11	12	36.00	3.00	0.93
12	12	51.96	4.33	0.43
Overall Total	144	38.34	3.20	0.66

3.3 Estimation of Transformed Missing Observations

Using (15)

$$Transformed\ value\ \hat{X}_{5,4} = 3.7062 + (-0.0320)[(5-1)12+4] + 0.0003[(5-1)12+4]^2 + (-0.0366)$$

$$= 2.8534$$

$$Transformed\ value\ \hat{X}_{7,9} = 3.7062 + (-0.0320)[(7-1)12+9] + 0.0003[(7-1)12+9]^2 + (-0.1922)$$
$$= 2.8903$$

$$Transformed\ value\ \hat{X}_{8,11} = 3.7062 + (-0.0320)[(8-1)12+11] + 0.0003[(8-1)12+11]^2 + (-0.3951)$$
$$= 2.9786$$

Estimation of Untransformed Incomplete Observations
Using (16)

$$Untransfomed\ value\ \hat{X}_{5,4} = e^{2.8534} = 17.3466 \approx 17$$

$$Untransfomed\ value\ \hat{X}_{7,9} = e^{2.8903} = 17.9987 \approx 18$$

$$Untransfomed\ value\ \hat{X}_{8,11} = e^{2.9786} = 19.6602 \approx 20$$

Table 6: Estimates of the transformed and untransformed missing observations

Missing Position	Transformed missing observations	Untransformed missing observations
$\hat{X}_{5,4}$	2.77	17.35
$\hat{X}_{7,9}$	3.69	18.00
$\hat{X}_{8,11}$	0.69	19.66

Table 7: Difference between trend parameters and seasonal indices with and without missing observations

parameter	With missing observations	Without missing observations	Differences
$\hat{a}$	3.7062	3.6881	0.0181
$\hat{b}$	-0.0302	-0.0286	0.0016
$\hat{c}$	0.0003	0.0003	0.0000
$\hat{S}_1$	1.4410	1.4346	0.0064
$\hat{S}_2$	-0.2443	-0.2507	0.0064
$\hat{S}_3$	-0.2983	-0.3046	0.0063
$\hat{S}_4$	-0.0366	0.1009	0.1375
$\hat{S}_5$	-0.0798	-0.0862	0.0064
$\hat{S}_6$	-0.0555	0.0619	0.1174
$\hat{S}_7$	-0.3158	-0.3222	0.0064
$\hat{S}_8$	-0.3177	-0.3241	0.0064
$\hat{S}_9$	-0.1922	-0.2246	0.0324
$\hat{S}_{10}$	-0.5083	-0.5147	0.0064
$\hat{S}_{11}$	-0.3951	-0.3694	0.0257
$\hat{S}_{12}$	0.9294	0.9229	0.0065

4. CONCLUDING REMARKS

This Study has discussed the Buys-Ballot estimates of quadratic trend cycle component and seasonal indices and effect of incomplete observations in descriptive in time series analysis and admits additive model. The methods adopted in this study are: 1) Buys-Ballot procedure for time series decomposition 2) Mean imputation 3) Regression imputation. Results show that the difference between parameters of trend with and without missing observations has insignificant effect, but significant in the seasonal indices of Buys-Ballot Table. The estimated incomplete observations are seventeen in April 2013, eighteen in September 2015 and twenty in November 2016.

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Appendix A: Buys-Ballot table for the actual data on number of road traffic accidents with incomplete observations (2009-2018)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2009	36	8	18	30	26	26	38	26	18	20	22	46	26.2	105.8
2010	68	14	16	12	18	48	14	34	32	20	38	70	32.0	425.5
2011	98	8	6	18	46	16	36	28	16	30	24	70	33.0	731.3
2012	106	20	4	20	26	42	4	48	36	30	20	122	39.8	1386.5
2013	86	24	30	-	40	4	26	8	14	50	12	52	30.2	555.2
2014	42	10	20	64	6	24	24	16	56	24	68	48	33.5	452.5
2015	68	12	12	8	10	12	10	2	-	2	2	38	18.0	408.0
2016	80	16	12	12	12	30	6	14	10	8	-	54	21.3	531.2
2017	154	22	28	30	18	14	24	20	6	6	16	144	40.2	2644.0
2018	254	72	64	78	66	86	54	62	78	40	50	314	101.5	7595.0
$\bar{X}_{.j}$	99.2	20.6	21.0	28.8	26.8	30.2	23.6	25.8	30.6	23.0	25.4	95.8		
$\sigma^2_{.j}$	4085.5	358.3	298.9	557.5	352.2	567.5	251.4	336.4	520.9	232.2	442.7	7080.4		

Appendix B: Buys-Ballot table for the transformed data on number of road traffic accidents with incomplete observations (2009-2018)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2009	3.58	2.08	2.89	3.40	3.26	3.26	3.64	3.26	2.89	3.00	3.09	3.83	3.18	0.21
2010	4.22	2.64	2.77	2.48	2.89	3.87	2.64	3.53	3.47	3.00	3.64	4.25	3.28	0.39
2011	4.59	2.08	1.79	2.90	3.83	2.77	3.58	3.33	2.77	3.40	3.00	4.25	3.21	0.66
2012	4.68	3.00	1.39	3.00	3.26	3.74	1.39	3.87	3.58	3.40	3.00	4.80	3.25	1.12
2013	4.45	3.18	3.40	-	3.69	1.37	3.26	2.08	2.64	3.91	2.48	3.95	3.10	0.76
2014	3.74	2.30	3.00	4.16	1.79	3.18	3.18	2.77	4.03	3.18	4.22	3.87	3.28	0.57
2015	4.22	2.48	2.48	2.08	2.30	2.48	2.30	0.69	-	0.69	0.69	3.64	2.31	1.39
2016	4.38	2.77	2.48	2.48	2.48	3.40	1.79	2.64	2.30	2.08	-	3.99	2.63	0.95
2017	5.04	3.09	3.33	3.40	2.89	2.64	3.18	3.00	1.79	1.79	2.77	4.97	3.16	1.01
2018	5.54	4.28	4.16	4.36	4.19	4.45	3.99	4.12	4.36	3.69	3.91	5.75	4.40	0.38
$\bar{X}_{.j}$	4.44	2.79	2.77	3.10	3.06	3.12	2.89	2.93	3.15	2.81	2.77	4.33	3.28	0.39
$\sigma^2_{.j}$	0.33	0.43	0.64	0.54	0.55	0.73	0.71	0.98	0.65	0.99	1.47	0.43	3.20	0.67

Appendix C: Buys-Ballot table for the actual data on number of road traffic accidents with complete observations (2009-2018)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2009	36	8	18	30	26	26	38	26	18	20	22	46	26.2	105.8
2010	68	14	16	12	18	48	14	34	32	20	38	70	32.0	425.5
2011	98	8	6	18	46	16	36	28	16	30	24	70	33.0	731.3
2012	106	20	4	20	26	42	4	48	36	30	20	122	39.8	1386.5
2013	86	24	30	16	40	4	26	8	14	50	12	52	30.2	555.2
2014	42	10	20	64	6	24	24	16	56	24	68	48	33.5	452.5
2015	68	12	12	8	10	12	10	2	40	2	2	38	18.0	408.0
2016	80	16	12	12	12	30	6	14	10	8	2	54	21.3	531.2
2017	154	22	28	30	18	14	24	20	6	6	16	144	40.2	2644.0
2018	254	72	64	78	66	86	54	62	78	40	50	314	101.5	7595.0
$\bar{X}_{.j}$	99.2	20.6	21.0	28.8	26.8	30.2	23.6	25.8	30.6	23.0	25.4	95.8		
$\sigma^2_{.j}$	4085.5	358.3	298.9	557.5	352.2	567.5	251.4	336.4	520.9	232.2	442.7	7080.4		

Appendix D: Buys-Ballot table for the transformed data on number of road traffic accidents with complete observations (2009-2018)

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2009	3.58	2.08	2.89	3.40	3.26	3.26	3.64	3.26	2.89	3.00	3.09	3.83	3.18	0.21
2010	4.22	2.64	2.77	2.48	2.89	3.87	2.64	3.53	3.47	3.00	3.64	4.25	3.28	0.39
2011	4.59	2.08	1.79	2.90	3.83	2.77	3.58	3.33	2.77	3.40	3.00	4.25	3.21	0.66
2012	4.68	3.00	1.39	3.00	3.26	3.74	1.39	3.87	3.58	3.40	3.00	4.80	3.25	1.12
2013	4.45	3.18	3.40	2.77	3.69	1.37	3.26	2.08	2.64	3.91	2.48	3.95	3.10	0.76
2014	3.74	2.30	3.00	4.16	1.79	3.18	3.18	2.77	4.03	3.18	4.22	3.87	3.28	0.57
2015	4.22	2.48	2.48	2.08	2.30	2.48	2.30	0.69	3.69	0.69	0.69	3.64	2.31	1.39
2016	4.38	2.77	2.48	2.48	2.48	3.40	1.79	2.64	2.30	2.08	0.69	3.99	2.63	0.95
2017	5.04	3.09	3.33	3.40	2.89	2.64	3.18	3.00	1.79	1.79	2.77	4.97	3.16	1.01
2018	5.54	4.28	4.16	4.36	4.19	4.45	3.99	4.12	4.36	3.69	3.91	5.75	4.40	0.38
$\bar{X}_{.j}$	4.44	2.79	2.77	3.10	3.06	3.12	2.89	2.93	3.15	2.81	2.77	4.33	3.28	0.39
$\sigma^2_{.j}$	0.33	0.43	0.64	0.54	0.55	0.73	0.71	0.98	0.65	0.99	1.47	0.43	3.20	0.67

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